Q: A 100 kg object is pushed up a $30^{\circ}$ ramp at constant speed. If the coefficient of friction is 0.2 , how much force is required?

A: Begin by setting up Newton's second law in the x direction

$$
\begin{aligned}
& \sum F_{x}=F-f_{k}-W_{\|}=0 \\
& F-N \mu_{k}-m g \sin \theta=0 \\
& F-m g \mu_{k} \cos \theta-m g \sin \theta=0 \\
& F=m g \mu_{k} \cos \theta+m g \sin \theta \\
& =(100 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(0.2)\left(\cos 30^{\circ}\right)+(100 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)\left(\sin 30^{\circ}\right) \\
& \approx 660 \mathrm{~N}
\end{aligned}
$$


(Note that $N=m g \cos \theta$ can be found by setting up Newton's laws in the y-direction.)

Q: Supposing that the box is pushed $d=10 \mathrm{~m}$ up the ramp, calculate the work done by each of the forces (normal, applied, friction, weight).

A: (The work done by the normal force is easy to do; since it is acts at a direction perpendicular to the displacement, it does no work)

$$
\begin{aligned}
& W_{N}=m g \cos \theta \cdot d \cos 90^{\circ}=0 \mathrm{~J} \\
& W_{F}=(660 \mathrm{~N})(10 \mathrm{~m}) \cos 0^{\circ} \approx 6600 \mathrm{~J} \\
& W_{f}=\left(m g \mu_{k} \cos \theta\right)\left(d \cos 180^{\circ}\right) \approx(100 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(0.2)\left(\cos 30^{\circ}\right)(10 \mathrm{~m})(-1) \approx-1700 \mathrm{~J} \\
& W_{W}=(m g)\left(d \cos 120^{\circ}\right)=(100 \mathrm{~kg})\left(9.81 \mathrm{~m}^{2}\right)(10 \mathrm{~m})(-0.5) \approx-4900 \mathrm{~J}
\end{aligned}
$$

Notice that the work done by forces opposing the motion is negative. Also, the total work done by all the forces
$W_{\text {TOT }}=W_{N}+W_{F}+W_{f}+W_{W}=0 \mathrm{~J}+6660 \mathrm{~J}-1700+\mathrm{J}-4900 \mathrm{~J}=0 \mathrm{~J}$
is zero. This makes since because according to the work-energy theorem, if an object's kinetic energy does not change, no work is done on it (and vice versa).

